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SOME MANIFESTATIONS OF ORIENTED TRANSFORM ABILITY OF SHAPE-MEMORY ALLOYS

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A unique feature of shape-memory alloys is that they can undergo oriented transformation [1, 2]. The essence of this phenomenon is as follows. Let a sample which is under stress σ_{ij} cool in the direct martensite transformation temperature range (M_1, M_2). This gives rise to direct-transformation strain ε_{ij} whose deviator is coaxial to the applied stress deviator. At a certain intermediate temperature T_0 ($M_2 < T_0 < M_1$), the acting stress vanishes. With a further decrease in temperature, the unloaded sample continues to deform in the direction of the previously applied stress. This can be explained by the growth of martensite crystallites oriented by the previously acting stress [2]. This phenomenon has been much studied experimentally on one-dimensional samples under tension and torsion [3, 4] and was described by micromechanical constitutive equations for shape-memory alloys [5–7].

In this paper, a method of solving boundary-value problems of direct transformations with variable boundary conditions is proposed. The method makes it possible to describe more complicated cases of oriented transformation.

1. Constitutive equations for shape-memory alloys that model, using the micromechanical approach of [8], the process of nucleation and growth of thermoelastic martensite crystallites in an austenite matrix are given in [5–7]. For direct transformation from a fully austenite state, these constitutive equations have the form

$$\varepsilon_{ij} = \varepsilon^1_{ij} + \varepsilon^2_{ij} + \varepsilon^3_{ij}, \quad \varepsilon^{1\prime}_{ij} = \frac{\sigma'_{ij}}{2G}, \quad \varepsilon^1_{kk} = \frac{1}{K}\sigma_{kk}, \quad \varepsilon^3_{ij} = \alpha(T - T_0)\delta_{ij},$$
$$\frac{d\varepsilon^{2\prime}_{ij}}{dq} = c_0\sigma'_{ij} + a_0\varepsilon^{2\prime}_{ij}, \quad \frac{d\varepsilon^2_{kk}}{dq} = \beta_0 + a_0\varepsilon^2_{kk}; \tag{1.1}$$

$$q = \sin\left(\frac{\pi}{2}\,\frac{T - M_1}{M_2 - M_1}\right), \tag{1.2}$$

where ε^1_{ij}, ε^2_{ij}, and ε^3_{ij} are the elastic, phase, and thermal strains, respectively; $T_0 \geqslant M_1$ is the temperature at which the thermal strain is considered equal to zero; σ_{ij} is the stress tensor; the prime denotes the deviator components; G and K are the shear and bulk moduli, respectively; α is the thermal-expansion coefficient; q is the volume portion of the martensite phase; and c_0, a_0, and β_0 are material constants whose values for titanium nickelide [3] and CuAlMnCo alloy [4] were determined in [6].

The solution of the differential equation for the phase-strain deviator is written as

$$\varepsilon^{2\prime}_{ij} = \int\limits_0^q K(q - \xi)\sigma'_{ij}(\xi)d\xi, \quad K(q) = c_0\exp(a_0 q).$$

Here is an evident similarity with the case of linear viscoelasticity, the parameter q playing the role of time [it is possible to use other kernels, e.g., a power kernel $K(q) = c_0 q^m$)]. A fundamental difference of the model considered is that its kernel is an increasing function ($a_0 > 0$ and $m > 0$), i.e., shape-memory alloys can be treated, by virtue of the above-mentioned analogy, as media with strengthened rather than decayed memory

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about loading prehistory (with respect to q). This is connected with oriented transformation, which in ordinary viscoelastic media with decayed memory is not observed. The second difference is that the parameter q can both increase and decrease. In the case of decreasing q (inverse transform), completely different constitutive equations hold true [5–7].

For many shape-memory alloys, the volume portion of the martensite phase q depends not only on temperature but also on acting stresses. However, this dependence can be ignored in studies of the deformations caused by oriented transformation in which stresses are removed.

2. In solution of boundary-value problems, a Laplace transform with respect to the variable q is applied simultaneously to the constitutive equations, equilibrium equations, compatiblity conditions, and boundary conditions. In the image space, the constitutive equations for the image of the total strain take the form

$$\hat{\varepsilon}'_{ij} = \frac{1}{2\hat{G}(s)}\hat{\sigma}'_{ij}, \quad \hat{\varepsilon}_{kk} = \frac{1}{K}\hat{\sigma}_{kk} + \hat{\varepsilon}_0(s),$$

where the hat denotes a Laplace transform;

$$s \to q, \ \hat{\varepsilon}_0(s) = \frac{\beta_0}{s(s-a_0)} + 3F(s), \ F(s) \to \alpha(T - T_0), \ \hat{G}(s) = G\,\frac{s-a_0}{s-d}, \ d = a_0 - 2c_0G \tag{2.1}$$

(the arrow indicates a Laplace correspondence). Thus, the direct-transformation problem reduces to an elastic problem with initial volumetric strain $\hat{\varepsilon}_0(s)$.

This problem can be solved as follows. If generally variable, volume forces $F_i(t)$ and right-hand sides of both the force $T_i^0(t)$ $[\sigma_{ij}n_j = T_i^0(t)$ on $S_T]$ and kinematical $u_i^0(t)$ $[u_i = u_i^0(t)$ on $S_u]$ boundary conditions are given, these quantities are expressed as functions of monotonically decreasing temperature $T(t)$: $F_i = F_i(T)$, $T_i^0 = T_i^0(T)$, and $u_i^0 = u_i^0(T)$.

Using relation (1.2) we can represent these functions as functions of q, which is extended arbitrarily through the point $q = 1$ (provided that Laplace transforms of the relevant functions exist). The simplest extensions are usually chosen. For example, functions that are constant in the interval $0 < q < 1$ are considered constant over the entire axis $q > 0$. Then, Laplace transforms are determined: $F_i(q) \to \hat{F}_i(s)$, $T_i^0(q) \to \hat{T}_i^0(s)$, and $u_i^0(q) \to \hat{u}_i^0(s)$. These will be, respectively, the volume forces and the force and kinematical boundary conditions of the equivalent elastic problem. Finding a solution of this problem as a function of the shear modulus $\hat{G}(s)$, of the initial volumetric strains $\hat{\varepsilon}_0(s)$, and of the quantities $\hat{F}_i(s)$, $\hat{T}_i^0(s)$, and $\hat{u}_i^0(s)$ and performing an inverse Laplace transform, one can solve the original direct transformation problem.

In what follows we shall need Laplace transforms for the following elastic constants:

$$\hat{E}(s) = E\,\frac{s-a_0}{s-\beta}, \quad \hat{\nu}(s) = \nu\,\frac{s-\gamma}{s-\beta}, \quad \hat{D}(s) = D\,\frac{(s-a_0)(s-\beta)}{(s-d)(s-\delta)}. \tag{2.2}$$

where $\beta = a_0 - 2c_0E/3$, $\gamma = a_0 - Ec_0/(3\nu)$, $\delta = a_0 - c_0E/(3(1-\nu))$, and E, ν, and D are the Young modulus, Poisson ratio, and cylindrical stiffness, respectively. It is easy to see that for $0 < \nu \leqslant 0.5$, $a_0 > 0$, and $c_0 > 0$, the following inequalities hold:

$$a_0 > \delta \geqslant \beta \geqslant d \geqslant \gamma, \tag{2.3}$$

where $\delta = \beta = d = \gamma$ for incompressible materials ($\nu = 0.5$).

3. We consider a rod of constant cross section and length L made of a shape-memory alloy which is in the austenite state at temperature T_0. Let the rod be fixed at one end, and the other end be subjected to longitudinal displacement u_0 within the limits of elastic deformations and then fixed in the new position. The rod stretched in such a manner is cooled in the temperature range (M_1, M_2). This should result in stress relaxation, because phase deformation propagates toward the acting stress.

The solution of the corresponding elastic problem with initial volume strain ε_0 has the form $\sigma = E(\delta_0 - (1/3)\varepsilon_0)$, where $\delta_0 = u_0/L$. Replacing the elastic modulus and the bulk strain by the expressions of $\hat{\varepsilon}_0(s)$ and of $\hat{E}(s)$ from (2.1) and (2.2), for the stress image, we obtain

$$\hat{\sigma}(s) = E\Big[\frac{\delta_0(s-a_0)}{s(s-\beta)} - \frac{\beta_0}{3s(s-a_0)} - F(s) + \frac{2c_0E}{3}\,\frac{F(s)}{s-\beta}\Big].$$

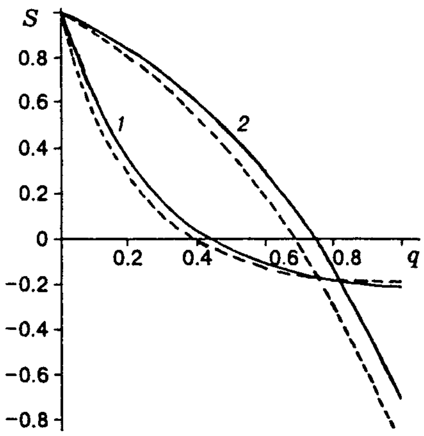

Fig. 1

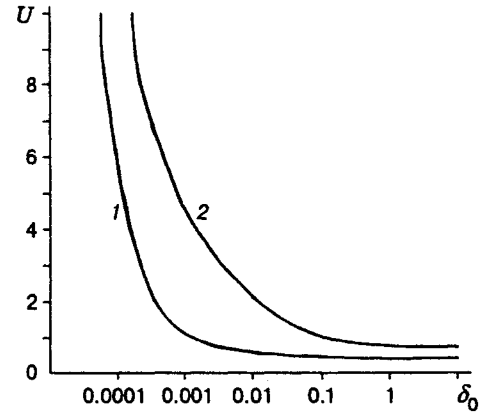

Fig. 2

Passing to the originals, one can find the temperature dependence of the stress:

$$\frac{\sigma}{\sigma_0} = \frac{1}{\beta}\Big(a_0 + \frac{\beta_0}{3\delta_0} - \lambda\frac{\varepsilon_2}{\delta_0}\Big) - \frac{\alpha(T - T_0)}{\delta_0} - \Big[\frac{1}{\beta}\Big(\lambda\Big(1 - \frac{\varepsilon_2}{\delta_0}\Big) + \frac{\beta_0}{3\delta_0}\Big) + \frac{6\lambda\varepsilon_1}{\pi\delta_0}I(q)\Big]\exp(\beta q). \tag{3.1}$$

Here $\sigma_0 = E\delta_0$, $\lambda = 2Ec_0/3$, $\varepsilon_1 = \alpha(M_1 - M_2)$, $\varepsilon_2 = \alpha(M_1 - T_0)$, and

$$I(q) = \int_0^q \arcsin(\tau)\exp(-\beta\tau)d\tau. \tag{3.2}$$

The curves of $S = \sigma/\sigma_0$ on q are shown in Fig. 1 by the solid curves for titanium nickelide [3] (curve 1) and for CuAlMnCo alloy [4] (curve 2). In the first case,

$$\begin{gathered} a_0 = 0.718, \quad c_0 = 0.243 \cdot 10^{-3}, \quad \beta_0 = 0.0011 \ \ [6, 7], \\ E = 28{,}000\,\text{MPa}, \quad \alpha = 0.6 \cdot 10^{-5} \ \ [9], \quad M_1 = 50\,^\circ\text{C}, \quad M_2 = 25\,^\circ\text{C}. \end{gathered} \tag{3.3}$$

For the copper-based alloy [4],

$$\begin{gathered} a_0 = 0.257, \quad c_0 = 0.152 \cdot 10^{-3}, \quad \beta_0 = 0.117 \cdot 10^{-3} \ \ [6, 7], \\ E = 7000\,\text{MPa}, \quad \alpha = 0.143 \cdot 10^{-4}, \quad M_1 = -27\,^\circ\text{C}, \quad M_2 = -34\,^\circ\text{C} \ \ [10]. \end{gathered} \tag{3.4}$$

In both cases, the initial stretching was carried out at temperature $T = M_1$ and initial strain $\delta_0 = 0.002$.

It is evident from the graphs that, as the temperature decreases, the stresses vanish at a certain value $q = q^*$, which depends on the material parameters and on the initial strain δ_0. For instance, for $\delta_0 = 0.002$ we have $q^* = 0.434$ for titanium nickelide and $q^* = 0.743$ for CuAlMnCo alloy. With a decrease in δ_0 the value of q^* also decreases.

Upon further cooling, compressive stresses arise in the initially stretched rod. This is the consequence of oriented transformation, because of which the phase-deformation rate is different from zero and positive, although for $q = q^*$ the stresses vanish. The ratio of the compressive stress for $q = 1$ to the initial tensile stress due only to the oriented transformation reaches 0.62 for CuAlMnCo alloy and 0.21 for titanium nickelide [these values were obtained from formula (3.1) in the limit $\delta_0 \to \infty$ in which the terms due to the volumetric phase and thermal strains vanish]. This effect is enhanced with a decrease in δ_0 (because of the volumetric and phase deformation and temperature change). On the one hand, the thermal compression decreases the compressive-stress magnitude, because of cooling, for $q > q^*$. On the other hand, for $q < q^*$, the thermal tensile stresses increase the subsequent effect of oriented transformation. Calculations show that the latter effect dominates. For $\delta_0 \to 0$, we have $|S(1)| \to \infty$.

Let now the rod be fixed at the other end, so that the rod is free to elongate by more than $L + u_0$, but it cannot become shorter than $L + u_0$. In this case, the boundary condition takes the form $u(L) = u_0$ for $\sigma < 0$ and $u(L) > u_0$ for $\sigma = 0$. For this problem, solution (3.1) is valid only for $q \leqslant q^*$. Upon further cooling, the rod elongates due to oriented transformation and volumetric phase strains and shortens due to thermal strains. It will be shown below that the elongation due to the first two factors exceeds the shortening due to the third factor. The rod will then be deformed in a stress-free state. The elastic strain is equal to zero, and the phase strain $\varepsilon^2(q)$ can be found by integrating Eqs. (1.1) for $\sigma_{ij} = 0$. As a result, the total strain can be determined from the formula

$$\varepsilon(q) = (\varepsilon^2(q^*) + \beta_0/3a_0)\exp[a_0(q - q^*)] - \beta_0/3a_0 + \alpha(T - T_0).$$

Expressing the value of the phase strain for $q = q^*$ in terms of the given initial displacement u_0 $[\varepsilon^2(q^*) = u_0/L - \alpha(T - T_0)]$, for the displacements of the free end of the rod for $q \geqslant q^*$, we obtain

$$u = u_0 \exp[a_0(q - q^*)] + L[\beta_0/3a_0 - \alpha(T - T_0)][\exp(a_0(q - q^*)) - 1]. \tag{3.5}$$

According to this dependence, the inequality $q > q^*$ holds true for $u > u_0$, i.e., the rod actually deforms in a stress-free state.

Figure 2 shows the relative additional displacement of $U = (u - u_0)/u_0$ versus the initial strain δ_0 for complete transformation ($q = 1$). Curve 1 corresponds to titanium nickelide, and curve 2 corresponds to CuAlMnCo alloy. $U \to \infty$ as $\delta_0 \to 0$ (because of the volumetric strain due to the phase transformation and the temperature term). As $\delta_0 \to \infty$, the quantity U tends asymptotically to the quantity $\exp[a_0(1 - q^*)] - 1$, which corresponds only to the phase strain due to shape change. One can see that the additional displacement caused by oriented transformation can exceed considerably the initial displacement u_0.

4. The problem of an infinite plane made of a shape-memory material with a round hole of radius a (planar stress) can be solved in a similar manner. A rigid round washer of radius $a + W_0$ is inserted into the hole. We assume that the outer boundary of the washer is fastened rigidly (welded) to the boundary of the hole. This corresponds to the boundary condition in the form of given radial displacement W_0 on the hole boundary. The solution of the corresponding elastic problem with initial volumetric strain ε_0 has the form

$$W = \frac{aW_0}{r} + \frac{1}{3}\varepsilon_0\Big(r - \frac{a^2}{r}\Big), \quad \sigma_r = -\sigma_\theta = 2G\frac{a}{r^2}\Big(W_0 - \frac{a\varepsilon_0}{3}\Big), \tag{4.1}$$

where r is the radius; W is the radial displacement; and σ_r and σ_θ are the radial and tangent stresses, respectively. Substituting the operators $\hat{G}(s)$ and $\hat{\varepsilon}_0(s)$ (2.1) for the modulus G and the volumetric strain ε_0 in the above expressions and moving to the originals, we obtain

$$W = \frac{aW_0}{r} + \Big(r - \frac{a^2}{r}\Big)\Big(\frac{1}{a_0}[\exp(a_0 q) - 1]\,\frac{\beta_0}{3} + \alpha(T - T_0)\Big),$$

$$\sigma_r = -\sigma_\theta = \sigma_0\Big\{\frac{a_0 - g\exp(qd)}{d} - \frac{1}{\delta_0}\Big[\frac{\beta_0/3 + g\varepsilon_1}{d}(\exp(qd) - 1) + \alpha(T - T_0) + \frac{2g\varepsilon_2}{\pi}\exp(qd)I(q)\Big]\Big\}. \tag{4.2}$$

Here $\sigma_0 = -2aGW_0/r^2$ is a function that describes the radial-stress distribution in the elastic problem, $g = 2Gc_0$, $\delta_0 = W_0/a$, and the integral $I(q)$ was defined above by relation (3.2).

The ratio $S = \sigma_r/\sigma_0$ versus q is shown in Fig. 1 by the dashed curves for titanium nickelide (3.3) (curve 1) and for CuAlMnCo alloy (3.4) (curve 2) ($\delta_0 = 0.002$). As can be seen from the graphs, the variation in stresses with an increase in q is qualitatively similar to the solution of the previous problem and differs from it quantitatively only slightly.

We now assume that the bonding between the washer and the hole boundaries is absent, i.e., the hole-boundary condition has the form $W = W_0$ for $\sigma_r < 0$ and $W > W_0$ for $\sigma_r = 0$. In this problem, solution (4.2) is true only for $q \leqslant q^*$. In accordance with the second formula of (4.1), all strain components vanish at any point of the semi-plane for $q = q^*$. Upon further cooling the material is free to deform in an unstressed state owing to oriented transformation. Proceeding as in the previous problem, for the hole-boundary radial displacement W, we have

$$W = W_0 \exp[a_0(q - q^*)] + a[\beta_0/3a_0 - \alpha(T - T_0)][\exp(a_0(q - q^*)) - 1]. \tag{4.3}$$

Formula (4.3) is derived from Eq. (3.5) by substituting the hole radius a for the rod length L. Nevertheless, the relative displacement W/W_0 in the problem of a plane with a hole is somewhat different from the value of u/u_0 of the rod problem for the same value of the relative initial displacement and the value of q, because the values of q^* are different in the two problems. But, qualitatively, the behavior of the solutions of both problems is the same. In particular, after a direct transformation in the half-plane the hole diameter can exceed considerably the washer diameter, so that their ratio tends to infinity as $\delta_0 \to 0$.

5. It is slightly more difficult to describe the experiment in which the phenomenon of oriented transformation was observed [1]. We consider a thin rectangular strip made of a shape-memory alloy in the austenite state and fixed at one end. The other end is displaced by W_0 upon one-side impact of a rigid support (the displacement can freely increase but not decrease). The band is then cooled in the direct martensite transformation temperature range. Vitaikin et al. [1] found that, starting with a certain intermediate temperature T_1, upon subsequent cooling, the strip moves away from the support, and the other end of the strip accomplishes an additional displacement in the same direction as W_0. The temperature T_1 does not depend on the initial deflection W_0. The total deflection W upon cooling to a fixed temperature is proportional to the initial deflection W_0 [1].

In solving the bending problem within the framework of the Kirchhoff–Love hypothesis, one can ignore the effect of volumetric, phase, and thermal deformations on the deflection. The solution of a cylindrical elastic bending problem for a rectangular strip fixed at one end and displaced by W_0 at the other end has the form

$$W = \frac{W_0}{2L^2}\Big(3x^2 - \frac{x^3}{L}\Big), \quad M_x = -\frac{3DW_0}{L^3}(L - x), \quad M_y = -\frac{3\nu DW_0}{L^3}(L - x), \quad P = \frac{3DW_0}{L^3}.$$

Here L is the strip length; x is the longitudinal coordinate; M_x and M_y are the bending moments; and P is the concentrated transverse force per unit of strip width which acts on the plate from the support for $x = L$. Applying a Laplace transform to the problem in which the position of the end of the plate is fixed from two sides and using expressions (2.2), for $\hat{D}(s)$ and $\hat{\nu}(s)$, we obtain

$$\frac{\widehat{M}_x(s)}{x - L} = \hat{P}(s) = \frac{3W_0D}{L^3}\frac{(s - a_0)(s - \beta)}{s(s - d)(s - \delta)}; \tag{5.1}$$

$$\frac{\widehat{M}_y(s)}{x - L} = \frac{3W_0\nu D}{L^3}\frac{(s - a_0)(s - \gamma)}{s(s - d)(s - \delta)}. \tag{5.2}$$

Transforming to originals, we have

$$P(q) = (3W_0D/L^3)f_1(q), \quad M_x(q) = P(q)(x - L); \tag{5.3}$$

$$f_1(q) = A\exp(dq) + B\exp(\delta q) + C; \tag{5.4}$$

$$A = \frac{d^2 - d(a_0 + \beta) + a_0\beta}{d(d - \delta)}, \quad B = \frac{\delta^2 - \delta(a_0 + \beta) + a_0\beta}{\delta(\delta - d)}, \quad C = \frac{a_0\beta}{d\delta}. \tag{5.5}$$

According to (5.3) and inequality (2.3), the force P decreases with increasing q. For some value of $q = q^*$ that is a root of the equation $f_1(q) = 0$ and does not depend on W_0, both the force P and the bending moment M_x vanish along with the stresses σ_x. After that, in the problem with a two-side fastening of one end of a plate, the force P, which is necessary to maintain the deflection W_0 constant, becomes negative and increases in absolute magnitude upon further cooling. The bending moment M_x and the corresponding normal stresses σ_x also change signs, so that tensile stresses σ_x emerge on the side of the plate facing displacement W_0 and compressive stresses emerge on the opposite side of the plate. It is clear that this effect is the result of oriented transformation. For the titanium nickelide with the above-mentioned characteristics, $q^* = 0.563$, and for the CuAlMnCo alloy $q^* = 0.817$.

Going to the originals in formula (5.2), for the bending moment M_y we obtain $M_y(q) =$

$(3W_0\nu D/L^3)f_2(q)(x-L)$, where the function $f_2(q)$ is calculated by formula (5.4), but one should substitute γ for the parameter β in expressions (5.5) for the coefficients. Using inequalities (2.3), one can show that the value of the function f_2 will decrease with increasing q, but more slowly than the corresponding values of f_1. Therefore, for the CuAlMnCo alloy, the transverse moment M_y remains positive while decreasing with increasing q. For titanium nickelide, both the moment M_y and the stresses σ_y vanish for $q = q^{**} = 0.687$, and they change signs upon further cooling. Since $q^{**} > q^*$, stress relaxation will be not complete in the plate for any value of q.

The solution obtained above is true also for $0 \leqslant q \leqslant q^*$ for the problem with a one-sided fastening of the right-hand end of a plate. Solution for $q > q^*$ is complicated due to the fact that the nonvanishing stresses σ_y remain in the plate, and the stress-deviator components entering into the phase strain equation are not only nonvanishing but vary in the course of deformation. For this reason, a Laplace transformation is used to solve the problem with one-side fastening for $q > q^*$. To find the bending deflection for $q > q^*$, one must solve the problem of the action on the right-hand plate end of a force $P_1(q)$ that varies as $P_1(q) = (3W_0D/L^3)f_1(q)$ for $q < q^*$ and $P_1(q) = 0$ for $q > q^*$. The solution of the corresponding equivalent elastic problem for the deflection of the right-hand end has the form

$$\hat{W}(s) = L^3\hat{P}_1(s)/(3\hat{D}(s)). \tag{5.6}$$

The Laplace transform of the function $P_1(q)$ is found most simply using the definition

$$\hat{P}_1(s) = \frac{3W_0D}{L^3}\int_0^{q^*} f_1(q)\exp(-sq)dq$$

$$= \frac{3W_0D}{L^3}\,\frac{A\delta\exp(q^*d) + Bd\exp(q^*\delta) - C\delta d + Cs(d+\delta)}{s(d-s)(\delta-s)}\exp(-sq^*). \tag{5.7}$$

Substituting (5.7) into (5.6) and going to originals, we obtain

$$\frac{W}{W_0} = \frac{(a_0+\lambda)\exp[\beta(q-q^*)] - (\beta+\lambda)\exp[a_0(q-q^*)]}{a_0-\beta}, \tag{5.8}$$

$$\lambda = Ad\exp(q^*d) + B\delta\exp(q^*\delta).$$

The solution is considerably simplified for an incompressible material ($\nu = 0.5$):

$$W = W_0\exp[a_0(q-q^*)]. \tag{5.9}$$

Relation (5.9) is true if one considers oriented transformation in a bent beam rather than in a plate. Since, according to (5.8) or (5.9), the inequality $W > W_0$ is valid for $q > q^*$, the right-hand end of the plate actually moves away from the support. Since q^* does not depend on W_0, the temperature at which the departure begins is the same for all values of W_0. This is in agreement with the experimental results of [1]. According to the solutions obtained above, the deflection W for fixed q is actually proportional to W_0, as was observed in the experiment of [1].

Figure 3 shows the dependence of $\xi = W/W_0$ on q calculated by formula (5.8) for the titanium nickelide (curve 1) and for the CuAlMnCo alloy (curve 2). It is interesting that for the material constants typical of these alloys, the values calculated by formulas (5.8) and (5.9) differ in the third decimal sign.

6. We now consider the contact problem of a punch with a parabolic base [profile $y = x^2/(2R)$] pressed by a force P to a half-plane made of shape-memory material in the austenite state. The normal displacements of points of the half-plane and punch are the same and there are no tangential stresses between them. The system is cooled in the direct martensite transformation temperature region. It is required to find the variation in pressing force P that provides a constant contact area $2L$.

The elastic solution for the problem of pressure S under the punch has the form [11]

$$S = \Big(\frac{G}{1-\nu}\,\frac{L^2-2x^2}{2R} + \frac{P(q)}{\pi}\Big)(L^2-x^2)^{-0.5}, \tag{6.1}$$

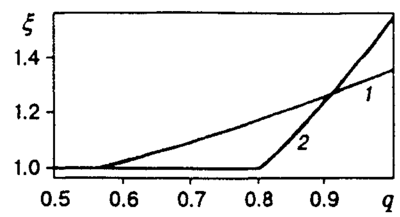

Fig. 3

where $P(q)$ is an unknown external force acting on the punch. Going to an equivalent elastic problem, we obtain

$$\hat{S}(s) = \Big(\frac{G}{1-\nu}\frac{(s-a_0)(s-\beta)}{s(s-d)(s-\delta)}\frac{L^2-2x^2}{2R} + \frac{\hat{P}(s)}{\pi}\Big)(L^2-x^2)^{-0.5},$$

where $\hat{P}(s)$ is the Laplace transform of the function $P(q)$. Going to the originals, we have

$$S(q,x) = \Big(\frac{G}{1-\nu} f_1(q)\frac{L^2-2x^2}{2R} + \frac{P(q)}{\pi}\Big)(L^2-x^2)^{-0.5},$$

where the function $f_1(q)$ is defined by formula (5.4). This problem with a varying load is simple to solve only because the term that is proportional to the force in solution (6.1) does not depend on the elastic constants. Taking into account the pressure finiteness conditions for $x = L$, we find the desired relation

$$P(q) = \frac{G}{1-\nu}\pi f_1(q)\frac{L^2}{2R}. \tag{6.2}$$

According to (6.2), upon cooling, the force required to keep the contact area constant decreases and vanishes for $q = q^*$, where q^* is a root of the equation $f_1(q) = 0$. For this value of q there is no force interaction between the punch and the half-plane, but the strain rate is nonvanishing because of oriented transformation. Thus, the force required to keep the contact area constant during cooling becomes negative, i.e., the material "draws" the punch into itself. Of course, this conclusion is true only in the formulation of problem considered, in which the points of the punch cannot move away from the corresponding points of the half-plane. In the problem of direct transformation with a constant force P acting on the punch, the dependence of the half-size of the contact area L on q can be found from Eq. (6.2). As a consequence, the contact area increases upon cooling and tends to infinity for $q \to q^*$. Again, this is the result of oriented transformation.

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